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SPATIAL INTERACTION OF STRONG DISCONTINUITIES IN A GAS*

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The spatial problem of the interaction of curved fronts of strong discontinuities during collision is examined for the system of gas-dynamic equations. In the case of regular interaction, an algorithm is indicated for the construction, and the existence of a piecewise-analytic solution of the problem in an exact formulation is proved. The series governing the solution converge in a certain neighbourhood of a two-dimensional surface γ_0 in the space $R^4(x, t)$, which is the intersection of surfaces of interacting discontinuities. It is shown that the solution cannot be piecewise-analytic in the neighbourhood of those points of γ_0 for which the normal velocity of the curve γ_{0t} with respect to the gas (a section through γ_0 by the plane $t = \text{const}$) is subsonic.

1. Formulation of the problem. For $t \in [-t_1, t_1]$ (t is the time), let an analytic solution $\mathbf{u} = \mathbf{u}_0(\mathbf{x}, t)$, $p = p_0(\mathbf{x}, t)$, $\rho = \rho_0(\mathbf{x}, t)$ of the system of gas-dynamics equations

$$\rho_t + \operatorname{div} \rho \mathbf{u} = 0, \quad (\rho u_l)_t + \operatorname{div}(\rho u_l \mathbf{u}) + (\nabla p)_l = 0 \tag{1.1}$$
$$(\rho(\varepsilon + {}^1/_2|\mathbf{u}|^2))_t + \operatorname{div} \rho \mathbf{u}(i + {}^1/_2|\mathbf{u}|^2) = 0 \quad (l = 1, 2, 3)$$

be known in the domain $\Omega \subset R^4(x, t)$ ($\mathbf{x} = (x_1, x_2, x_3) \in R^3$, $t \in R$) ($\mathbf{u} = (u_1, u_2, u_3)$ is the velocity vector, ρ is the density, p is the pressure, ε is the specific internal energy, and $i = \varepsilon + p\rho^{-1}$ is the specific enthalpy). The functions $\varepsilon = \varepsilon(v, p)$, $p = g(v, s)$ (here $v = \rho^{-1}$ and s is the entropy) that give the equation of state of the medium are analytic and satisfy the normal gas conditions /1/. The fronts of two strong discontinuities propagate over the background "null", where the surfaces of discontinuity $\Gamma_j \subset R^4(\mathbf{x}, t)$ and the solutions behind the fronts $\mathbf{u} = \mathbf{u}_j(\mathbf{x}, t)$, $p = p_j(\mathbf{x}, t)$, $\rho = \rho_j(\mathbf{x}, t)$ ($j = 1, 2$) are analytic. (The discontinuities are concentrated on the hypersurfaces Γ_j in the space $R^4(\mathbf{x}, t)$. Sections Γ_{jt} through these surfaces by the planes

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$t = \text{const}$ correspond to the instantaneous positions of the fronts of the discontinuities). At the time $t = 0$ the fronts first touch at the point Q. It is required to describe the interaction process for $t > 0$ if the motion of each of the discontinuities in the absence of another is known for $t \in [-t_1, t_1]$. The problem is examined in this formulation for the interaction of two shocks moving towards each other or for the incidence of a shock on a contact discontinuity.

Remarks. 1°. If for $t = 0$ the set $\Gamma_{10} \cap \Gamma_{20}$ contains its neighbourhood of Γ_{10} together with the point Q, then by virtue of analyticity Γ_{10} and Γ_{20} should coincide for $t = 0$. Then the problem arise regarding the decay of an arbitrary discontinuity on a curvilinear surface, as examined in /2/. We shall assume that Q is a single point of tangency of the fronts of interacting discontinuities for $t > 0$ (for $t > 0$ the tangent planes to the fronts do not coincide at points of intersection).

2°. The method proposed for constructing the solution can be used almost without change in cases when the tangency of the fronts at the initial time occurs at several points at once, or over a certain closed or open curve in $R^3(\mathbf{x})$ when tangency holds for $t > 0$ also. In particular, motion with plane-parallel symmetry can be considered when the tangency of the fronts occurs over the rectilinear of cylindrical surfaces.

2. Configuration of the singularities. The two-dimensional surface $\gamma_1 \subset R^3(\mathbf{x})$ which the line of intersection $\gamma_{0t} = \Gamma_{1t} \cap \Gamma_{2t}$ describes as t grows, is determined by the motion of the given front Γ_{jt}. Let γ_1 have no selfintersections and let one instant t exist for each point $\mathbf{x} \in \gamma_1$ such that $\mathbf{x} \in \gamma_{0t}$. Let us give the surface γ_1 and the appropriate points $\mathbf{x}$ of the times t by the parametric equations $\mathbf{x} = \mathbf{x}_0(\beta, \gamma)$, $t = t_0(\beta, \gamma)$; here $\mathbf{x}_0$, t_0 are analytic functions of the parameters β, γ that have a difference in length, $\mathbf{x}_{0\beta} \neq 0$, $\mathbf{x}_{0\gamma} \neq 0$, $\mathbf{x}_{0\beta} \times \mathbf{x}_{0\gamma} \neq 0$. If γ_1 is projected uniquely on the plane x_2x_3, then the variables x_2, x_3 can be taken as β, γ; the functions $x_{10}(x_2, x_3)$, $t_0(x_2, x_3)$ are determined from the equations of the surfaces Γ_j upon compliance with the conditions of the theorem on implicit functions and $(\mathbf{x}_0(\beta, \gamma) = (x_{10}(\beta, \gamma), \beta, \gamma))$. We will consider this method of parametrization basic although further constructions are applicable for other parametrizations also. A two-dimensional surface $\gamma_0 = \Gamma_1 \cap \Gamma_2 \subset R^4(\mathbf{x}, t)$ is given by the equations $\mathbf{x} = \mathbf{x}_0(\beta, \gamma)$, $t = t_0(\beta, \gamma)$.

The shock, contact discontinuities, configurations and waves centered at γ_0 /3/ are determined in the first stage of the construction of the solution. Relationships on the shocks and the centered waves, the contact discontinuity, and the conditions for passage of the surfaces of discontinuity through γ_0 in the case of a regular interaction are used here.

It is convenient to convert the Hugoniot relations on the shock front to a form containing the known vectors of the coordinate basis of the surface $\gamma_0: \boldsymbol{\vartheta}_1 = (x_{1\beta}, x_{2\beta}, x_{3\beta}, t_\beta)$, $\boldsymbol{\vartheta}_2 = (x_{1\gamma}, x_{2\gamma}, x_{3\gamma}, t_\gamma)$ (the derivatives are calculated on γ_0 and the index 0 is omitted). To do this, we use the relationships /4/

$$\mathbf{x}_\beta \mathbf{n} = t_\beta D_n, \quad \mathbf{x}_\gamma \mathbf{n} = t_\gamma D_n \tag{2.1}$$

where $\mathbf{n}$ is the normal to the shock front, and D_n is the velocity of motion of the front in the normal direction. As a consequence of the relationships on the shock we obtain /4/

$$\begin{gathered}[a] = [b] = 0, \quad \varepsilon(v, p) - \varepsilon(v_j, p_j) = {}^1/_2 (p + p_j)(v_j - v) \\ v_n = D_n - u_n = v (p - p_j)^{1/2} (v_j - v)^{-1/2} (u_n = (\mathbf{un})) \\ a = (\mathbf{u}\mathbf{x}_\beta) - t_\beta (i + 2^{-1} |\mathbf{u}|^2), \quad b = (\mathbf{u}\mathbf{x}_\gamma) - t_\gamma (i + {}^1/_2 |\mathbf{u}|^2)\end{gathered} \tag{2.2}$$

where $[f]$ means the jump in f during passage through the discontinuity. The quantities $a = (\mathbf{U}\boldsymbol{\vartheta}_1)$, $b = (\mathbf{U}\boldsymbol{\vartheta}_2)$ are covariant components of the vector function $\mathbf{U} = (u_1, u_2, u_3, -(i + {}^1/_2 |\mathbf{u}|^2))$ (on changing the parametrization β, γ by β', γ' we obtain a', b' which are obtained from a, b by a tensor transformation law). The conservation of the projection of the above-mentioned vector-function on the tangent plane to γ_0 during passage through the discontinuity follows from (2.2).

We define the concept of the normal velocity of the curve γ_{0t} moving with time in $R^3(\mathbf{x})$. We consider the normal plane to γ_{0t} at the point $A \in \gamma_{0t}$. Let B be the point of intersection of $\gamma_{0(t+\Delta t)}$ and the normal plane. The vector $\mathbf{N} = \lim_{\Delta t \to 0} \mathbf{AB}\,(\Delta t)^{-1}$ is called the normal velocity of γ_{0t} at point A. The difference between the normal velocity of γ_{0t} and the projection of the gas velocity vector on the normal plane is called the normal velocity of the curve of γ_{0t} relative to the gas. A simple calculation shows that the normal velocity of γ_{0t} agrees with the vector $(\mathbf{k} \times \mathbf{m})\,|\mathbf{k}|^{-2}$, while the normal velocity with respect to the gas agrees with the vector $\mathbf{w}$; $\mathbf{w} = (\mathbf{q} \times \mathbf{k})\,|\mathbf{k}|^{-2}$. Here $\mathbf{k} = t_\beta \mathbf{x}_\gamma - t_\gamma \mathbf{x}_\gamma$ is a vector directed along the tangent to γ_{0t}, $\mathbf{m} = \mathbf{x}_\beta \times \mathbf{x}_\gamma$ is a vector directed along the normal to γ_1, and $\mathbf{q} = \mathbf{m} + \mathbf{k} \times \mathbf{u}$ is a vector directed along the normal to the contact characteristic passing through γ_0 /4/.

We introduce φ, the angle between the vectors $\mathbf{w}$ and $-\mathbf{N}$

$$\cos\varphi = (\mathbf{q} \cdot \mathbf{m})\,|\mathbf{q}|^{-1} |\mathbf{m}|^{-1}, \quad \sin\varphi = |\mathbf{k}|\,(\mathbf{um})\,|\mathbf{q}|^{-1} |\mathbf{m}|^{-1} \tag{2.3}$$

The relationship /4/

$$\varphi-\varphi_j=\pm\arcsin\frac{|\mathbf{k}|}{|\mathbf{q}_j|}\left[\frac{(p-p_j)(v_j-v-v_j^2|\mathbf{q}_j|^{-2}|\mathbf{k}|^2(p-p_j))}{1-(p-p_j)(v_j+v)|\mathbf{k}|^2|\mathbf{q}_j|^{-2}}\right]^{1/2} \tag{2.4}$$

(here $\varphi-\varphi_j$ corresponds to the quantity θ in /4/) is obtained for the angle of rotation of the vector $\mathbf{w}$ on passing through the discontinuity, and agrees formally with the equation of the shock polar in the plane of the variables, pressure-slope of the velocity vector considered in the theory of indirect compression shocks. (The absolute value of the normal velocity vector γ_{0t} with respect to the gas ahead of the front corresponds to the absolute value of the free stream velocity vector and the subscript j refers to the state ahead of the front). Certain properties of the shock polar in a normal gas were studied in /5/.

The relationship (2.4) holds even at the point Q, where $t_\beta=t_\gamma=0$ ($t_0(\beta,\gamma)$ reaches a minimum on γ_1 at the point Q) and both sides of the equation vanish. But (2.4) can be written in the form $|\mathbf{k}||\mathbf{m}|^{-1}\Phi(\beta,\gamma)=0$, where Φ is continuous in the neighbourhood of the point where $t_\beta=t_\gamma=0$, consequently, it follows from (2.4) that $\Phi=0$ on γ_0 (at points where $|\mathbf{k}|\neq 0$ it is possible to separate $|\mathbf{k}||\mathbf{m}|^{-1}$ into factors, and $\Phi=0$ in continuity at the point Q).

The equality $\Phi=0$ is equivalent to the following

$$\sigma-\sigma_j=\pm\frac{|\mathbf{m}|}{|\mathbf{k}|}\arcsin\frac{|\mathbf{k}|}{|\mathbf{q}_j|}\left[\frac{(p-p_j)(v_j-v-v_j^2|\mathbf{q}_j|^{-2}|\mathbf{k}|^2(p-p_j))}{1-(p-p_j)(v_j+v)|\mathbf{k}|^2|\mathbf{q}_j|^{-2}}\right]^{1/2} \tag{2.5}$$

where $\sigma=|\mathbf{m}||\mathbf{k}|^{-1}\varphi$; as $|\mathbf{k}|\to 0$ we have $\sigma=|\mathbf{m}||\mathbf{k}|^{-1}\arcsin\{|\mathbf{k}|(\mathbf{u}\cdot\mathbf{m})\times|\mathbf{q}|^{-1}|\mathbf{m}|^{-1}\}$. Here and in (2.5), as $t_\beta^2+t_\gamma^2\to 0$

$$|\mathbf{k}|^{-1}\arcsin|\mathbf{k}|f=f+\sum_{l=1}^{\infty}\frac{(2l)!\,|\mathbf{k}|^{2l}f^{2l+1}}{2^{2l}(l!)^2(2l+1)}$$

Consequently (2.5) goes over into the following equation at the point Q

$$\sigma-\sigma_j=\pm(p-p_j)^{1/2}(v_j-v)^{1/2} \tag{2.6}$$

which is actually the equation of the $(p,\ u_n)$-pattern of the shocks /1, 6/ ($\sigma=u_n=(\mathbf{um})|\mathbf{m}|^{-1}$ for $t_\beta=t_\gamma=0,\ \mathbf{n}=\mathbf{m}\,|\mathbf{m}|^{-1}$). For $|\mathbf{k}|\neq 0$ the quantity σ equals the arc length on a circle of radius $|\mathbf{m}|\,|\mathbf{k}|^{-1}$ (the absolute value of the normal velocity γ_{0t}), shrunk by the angle φ taken with the sign $\operatorname{sgn}\varphi$. Exactly like the angle $\varphi-\varphi_j$ the quantity $\sigma-\sigma_j$ characterizes the rotation of the vector $\mathbf{w}$ during passage through a discontinuity, but it is more convenient for analysing the relationships on the discontinuity since $\sigma\not\equiv 0$ for $t_\beta=t_\gamma=0$ unlike φ.

It has been shown /3/ that the limit values on γ_0 of the fundamental quantities on both sides of the wave centered on γ_0 are connected by the equations

$$[a]=[b]=0,\ \ [s]=0,\ \ [\sigma\pm H(p,\ s,\ (|\mathbf{q}|^2+2i\,|\mathbf{k}|^2)\,|\mathbf{m}|^{-2})]=0 \tag{2.7}$$

$$H(p,s,\xi)=\int_0^p\frac{[\xi-|\mathbf{k}|^2|\mathbf{m}|^{-2}(2i+c^2)(p',s)]^{1/2}\,dp'}{\rho(p',s)\,c(p',s)\,(\xi-2|\mathbf{k}|^2|\mathbf{m}|^{-2}i(p',s))},\quad c^2=-v^2g_v(v,s)$$

($\mathbf{q}$ corresponds here to the vector $\mathbf{v}$ in /3/, and the quantity θ in /3/ is connected with σ by the relationship $\theta=|\mathbf{m}|^{-1}\sigma$). Using the fact that the vector $\mathbf{q}$ is directed along the normal to the front of the contact discontinuity /4/, the relationships on the contact discontinuity can be converted to the form

$$[p]=0,\ \ [\sigma]=0 \tag{2.8}$$

If the normal to the wave front is directed towards the state ahead of the front, then as can be shown for $(\mathbf{nq})>0$ the plus sign must be selected in (2.5) and the minus sign in (2.7) (the opposite for $(\mathbf{nq})<0$). We call the waves turned to the right (to the left) if $(\mathbf{nq})>0$ ($(\mathbf{nq})<0$).

The method of determining the configuration of the singularities on γ_0 is analogous to the method of the (p,u)-pattern for solving problems about the decay of an arbitrary discontinuity /1, 6/. The (p,σ)-patterns of transitions are examined at each point γ_0: for $p\geqslant p_j$ these curves are given by (2.5), while for $p\leqslant p_j$ they are given by

$$\sigma\mp H(p,s,(|\mathbf{q}_j|^2+2i_j|\mathbf{k}|^2)|\mathbf{m}|^{-2})=\sigma_j+H(p_j,s_j(|\mathbf{q}_j|^2+2i_j|\mathbf{k}|^2)|\mathbf{m}|^{-2})$$

Of the two states *1* and *2*, we call that gas state right towards which the vector $\mathbf{q}$ (coincident with $\mathbf{m}$ at $t=0$) is directed. A (p,σ)-pattern of the transitions is constructed for each point of the surface γ_0 such that from the points (p_j,σ_j) corresponding to the right state, a (p,σ)-pattern of transitions turned to the right results, while from the points corresponding to the left state, a pattern of transitions turned to the left results. Gas states that can be related by a contact discontinuity (in conformity with (2.8)) correspond to the points of intersection of the (p,σ)-patterns.

We assume that the equations of state of the gas satisfy the condition

$$(p + vg_v)\ \varepsilon_p + pv \leqslant 0 \tag{2.9}$$

This inequality ensures that the "supersonic" portions of the (p, σ)-patterns are monotonic, i.e., the σ depends monotonically on p (on the supersonic portion $|\mathbf{q}|^2 = |\mathbf{q}_j|^2 - (p - p_j)\ (v_j + v)\ |\mathbf{k}|^2 > |\mathbf{k}|^2 c^2$). By virtue of the above-mentioned connection, the corresponding properties of the (p, σ)-pattern are obtained from the properties of the shock polar studied in /5/. The configuration is defined uniquely at the point Q (by virtue of (2.9) the curves (2.6) are monotonic for all values of p and the point of intersection of the patterns is unique). The patterns drawn from the points $(p_j(\beta, \gamma), \sigma_j(\beta, \gamma))$ (where the parameters γ, β correspond to points on γ_0 that are close to Q) can have two or more points of intersection. (We show in Fig.1 (p, σ)-patterns corresponding to shock interaction in a polytropic gas). From continuity considerations for the solution behind the shock front, the configuration of singularities is determined in this case by the point of intersection with minimum p (point 3 in Fig.1).

The appearance of other points of intersection is actually possible for the values $p \geqslant \min(p_1^*, p_2^*)$, where p_j^* are the coordinates of points on the p axis where the (p, σ)-patterns have a vertical tangent. The values of p_j^* are determined from the equations

$$\frac{|\mathbf{q}_j|^2}{|\mathbf{k}|^2} = v_j (p - p_j) \frac{v_j + v - v'(p - p_j)}{v_j - v - v'(p - p_j)}$$

where $v = v(p, p_j, v_j)$ (by virtue of the equations of the Hugoniot adiabat, and $v' = \partial v(p, p_j, v_j)/\partial p$. The left side of the equality tends to infinity as $|\mathbf{k}| \to 0$, and then $p_j^* \to \infty$ $(v_j - v - v'(p - p_j)) \neq 0$ by virtue of (2.9)(/5/). Therefore, the remaining points of intersection are far from the point found $(p_3(Q), \sigma_3(Q))$ for small t_β, t_γ. The values of (p_3, σ_3), connected in a continuous manner with $(p_3(Q), \sigma_3(Q))$, are defined uniquely in the neighbourhood ω of the point Q on γ_0. The boundary of the neighbourhood ω is determined by the tangency condition of the (p, σ)- patterns drawn at the corresponding points $(p_j(\beta, \gamma), \sigma_j(\beta, \gamma))$ (the ambiguity in determining the configuration can occur during the merger of two roots, and this indeed corresponds to the boundary of ω). After having determined σ, p, v, a, b on γ_0, the vector $\mathbf{u}$ is restored by the formulas presented in /4/ and

$$\mathbf{u} = \Big\{|\mathbf{q}|\,|\mathbf{m}|^{-1}|\mathbf{k}|^{-1}\sin(|\mathbf{k}|\,|\mathbf{m}|^{-1}\sigma)\,\mathbf{m} + |\mathbf{m}|^{-2}(a\mathbf{x}_\gamma - b\mathbf{x}_\beta) \times \mathbf{m} + \frac{|\mathbf{q}|^2|\mathbf{k}|^{-2}\sin^2(|\mathbf{k}|\,|\mathbf{m}|^{-1}\sigma) + (a\mathbf{x}_\gamma - b\mathbf{x}_\beta)^2|\mathbf{m}|^{-2} + 2i}{1 + |\mathbf{q}|\,|\mathbf{m}|^{-1}\cos(|\mathbf{k}|\,|\mathbf{m}|^{-1}\sigma) + (\mathbf{k},\ b\mathbf{x}_\beta - a\mathbf{x}_\gamma)\,|\mathbf{m}|^{-2}} \cdot \frac{\mathbf{k}\times\mathbf{m}}{|\mathbf{m}|^2}\Big\} \tag{2.10}$$

Different configurations of shock and centered waves occur depending on which poritions of the patterns intersect at point 3 (similar to the problem of the decay of an arbitrary discontinuity). The criteria presented in /2/ can be utilized in determining the kind of configuration, depending on the data of the problem about the point Q.

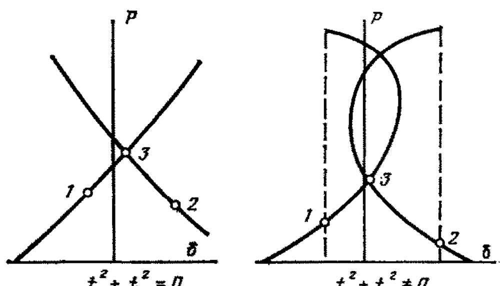

Fig.1

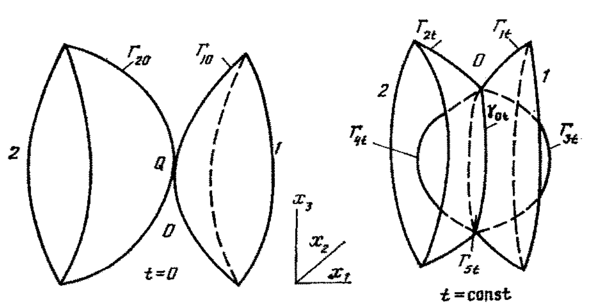

Fig.2

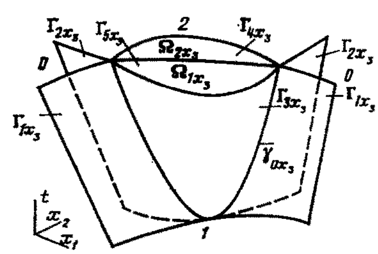

Fig.3

3. Configuration of two shocks. Let the data of the problem be such that for all $(\mathbf{x}, t) \in \omega_1 \subset \omega \subset \gamma_0$ the point of intersection of the (p, σ)-patterns lies on branches corresponding to shocks. It is required to find the surfaces of the reflected shocks Γ_3, Γ_4, the contact discontinuity surface Γ_5, and the solution of (1.1) in the domains Ω_j bounded by these surfaces.

The pattern in the sections $t = \text{const}$ corresponding to the given problem is displayed in Fig.2, and the pattern in the section $x_3 = \text{const}$ corresponding to the problem of flow with plane-parallel symmetry in Fig.3 (the flow characteristics are independent of x_3, see Remark 2°).

A change to the auxiliary variables $\tau, \alpha, \beta, \gamma$ in the domains Ω_j is made in constructing the solution so that the unknown boundaries are fixed in the new variables (τ, α have the dimensions of time, and β, γ the dimensions of length), then after the problem has been solved in the fixed domain, the possibility of transferring to the initial variables is then proved. Let the vector $\mathbf{m}$ be directed towards the state 1 for $t = 0$. The replacement of the variables is given by the relationships $t = \tau + \alpha + t_0(\beta, \gamma)$, $\mathbf{x} = \mathbf{x}(\tau, \alpha, \beta, \gamma)$ where the vector $\mathbf{x}(\tau, \alpha, \beta, \gamma)$ is determined by solving the equations

$$\mathbf{y}_\alpha = \mathbf{u} \pm \frac{v_n \mathbf{q}_*}{(|\mathbf{q}_*|^2 - v_n^2 |\mathbf{k}_*|^2)^{1/2}}, \quad \mathbf{y}|_{\alpha=0} = \mathbf{x}_0(\beta, \gamma) \tag{3.1}$$

$$\mathbf{x}_\tau = (\mathbf{um}) |\mathbf{q}|^{-2} \mathbf{q} - V^2 |\mathbf{m}|^{-1} |\mathbf{q}|^{-1} (\mathbf{k} \times \mathbf{q}), \quad (\mathbf{x} - \mathbf{y})|_{\tau=0} = 0$$

$$v_n = v(p - p_j)^{1/2} (v_j - v)^{-1/2}$$

Here V is a positive constant such that on γ_0 $|\mathbf{u}| < V$, $\mathbf{q}_* = (\mathbf{y}_\beta - t_\beta \mathbf{u}) \times (\mathbf{y}_\gamma - t_\gamma \mathbf{u})$, $\mathbf{k}_* = t_\beta \mathbf{y}_\gamma - t_\gamma \mathbf{y}_\beta$ the plus sign corresponds to the domain Ω_1 and the minus to the domain Ω_2. The normal to the shock front can be calculated in the form /4/

$$\mathbf{n} = (\pm (|\mathbf{q}|^2 - v_n^2 |\mathbf{k}|^2)^{1/2} \mathbf{q} + v_n (\mathbf{k} \times \mathbf{q})) |\mathbf{q}|^{-2} \tag{3.2}$$

Consequently $\mathbf{x}_\alpha \mathbf{n} = t_\alpha D_n$ for $\tau = 0$ and $(\mathbf{x}_\tau - \mathbf{u}) \mathbf{q} = 0$ everywhere. This means that the plane $\tau = 0$ corresponds to the shock surface and the plane $\alpha = \text{const}$ to the contact characteristics; in particular, Γ_5 corresponds to $\alpha = 0$ /4/. The Jacobian of the passage to the new variables for $\tau = 0$ is calculated in the form

$$I|_{\tau=0} = (\mathbf{x}_\tau - \mathbf{x}_\alpha) [\mathbf{m} + \mathbf{k} \times \mathbf{x}_\tau] = \mp (v_n |\mathbf{q}|^2 J^{-1}) \times (|\mathbf{q}|^2 - v_n^2 |\mathbf{k}|^2)^{-1/2}|_{\tau=0}$$

and is different from zero on γ_0, at least, since the inequality $J^{-1} \geqslant 1/2$ holds for the quantities $J^{-1} = (\mathbf{qm}) |\mathbf{q}|^{-2} + V^2 |\mathbf{k}|^2 |\mathbf{m}|^{-1} |\mathbf{q}|^{-1}$

After introducing new variables in Ω_j the $\mathbf{x}_\beta$, $\mathbf{x}_\gamma$, t_β, t_γ are determined, and consequently the quantities σ, a, b can be introduced in Ω_j by the same formulas as in Sect.2. For $\tau = 0$ the relationships (2.1) are satisfied since the vectors $\vartheta_1 = (x_{1\beta}, x_{2\beta}, x_{3\beta}, t_\beta)$, $\vartheta_2 = (x_{1\gamma}, x_{2\gamma}, x_{3\gamma}, t_\gamma)$ lie in the tangent plane to the shock surface. It can be shown by utilizing (2.1) that the boundary conditions on the shock front can be written in the form (2.2), (2.5) (as in Sect.2.

System (1.1) is converted to new variables in the domain Ω_1

$$a_\tau = e_1 \mathbf{v}_\beta + e_2 \mathbf{v}_\gamma + e_3, \quad b_\tau = e_4 \mathbf{v}_\beta + e_5 \mathbf{v}_\gamma + e_6, \quad s_\tau = e_7 s_\beta + e_8 s_\gamma \tag{3.3}$$

$$d\mathbf{h}_\tau = \mathbf{h}_\alpha + e_9 \mathbf{v}_\beta + e_{10} \mathbf{v}_\gamma + e_{11} (\mathbf{x}_\alpha)_\beta + e_{12} (\mathbf{x}_\alpha)_\gamma + e_{13}$$

$$\mathbf{h} = \begin{pmatrix} \sigma \\ p \end{pmatrix}; \quad d = \frac{(\mathbf{x}_\tau - \mathbf{x}_\alpha)\mathbf{q}}{|\mathbf{q}|^2} \begin{Vmatrix} (\zeta, \mathbf{q}) & (c^{-2} - |\mathbf{q}|^{-2} |\mathbf{k}|^2) J |\mathbf{m}| \rho^{-1} \\ \rho |\mathbf{q}|^2 |\mathbf{m}|^{-1} J & (\zeta \mathbf{q}) \end{Vmatrix}$$

Here $\mathbf{v}$ is the notation of the vector solution whose components are the quantities a, b, s, $\sigma, p, x_{i\beta}, x_{i\gamma}, x_i$ $(i = 1, \ldots, 3)$; e_l are scalar, vector, and matrix functions of the variables $\mathbf{v}$, $\mathbf{x}_\alpha$, β, γ and $\zeta = I^{-1} (\mathbf{m} + \mathbf{k} \times \mathbf{x}_\alpha)$. The derivatives a_α, b_α are not in system (3.3); the selection of a, b as the desired functions is explained by this circumstance and the simplicity of the boundary conditions (2.2). In the domain Ω_2 the system of equations has a form analogous to (3.3), taking the changes when determining the change of variables into account. The desired functions and the coefficients of the equations in Ω_2 will be denoted by corresponding capital letters: $A, B, S, \Sigma, \ldots, E_l, D$. The question of the existence of a solution of the problem will be solved by constructing a Taylor series for the solution in the neighbourhood of the points ω_1 and proving their convergence.

We will show that the equations and boundary conditions permit a unique determination of all the derivatives of the solution at an arbitrary point of a certain subdomain of ω_1. To this end, we transform the boundary conditions of the form (2.5) by extracting the linear part in σ and p at the point $N \in \omega_1 \subset \gamma_0$

$$\boldsymbol{\lambda} \begin{pmatrix} \sigma \\ p \end{pmatrix} = f_1(p, p_1, v_1, \sigma_1, |\mathbf{k}|^2, |\mathbf{q}_1|, |\mathbf{m}|) \tag{3.4}$$

$$\boldsymbol{\Lambda} \begin{pmatrix} \Sigma \\ P \end{pmatrix} = F_1(P, p_2, v_2, \sigma_2, |\mathbf{k}|^2, |\mathbf{q}_2|, |\mathbf{m}|)$$

$$\left(\boldsymbol{\lambda} = (1, \lambda) = \left(1 \left(\frac{\partial \sigma}{\partial p}\right)_N\right), \quad \boldsymbol{\Lambda} = (1, \Lambda) = \left(1 \left(\frac{\partial \Sigma}{\partial P}\right)_N\right)\right.$$

$$\left(\frac{\partial f_1}{\partial p}\right)_N = \left(\frac{\partial F_1}{\partial P}\right)_N = 0)$$

The last equations of system (3.3) are written in the form

$$d_N \mathbf{h}_\tau = \mathbf{h}_\alpha + \boldsymbol{\varphi}, \quad D_N \mathbf{H}_\tau = \mathbf{H}_\alpha + \boldsymbol{\Phi} \tag{3.5}$$

($d_N = (d)_N$, $D_N = (D)_N$ in $\boldsymbol{\varphi}$ and $\boldsymbol{\Phi}$ are referred to the remaining terms of the equations; this transformation separates the principal part in the derivatives $\mathbf{h}_\tau$, $\mathbf{h}_\alpha$ at the point N).

In clarifying the question of the solvability of differentiated equations and boundary conditions with respect to the derivatives of the desired functions in the variables τ and α the relationships (3.4) and (3.5) play an important part in connected with the fact that the boundary conditions for σ and p are given for $\tau = 0$ and $\alpha = 0$. We will write the differential consequences (3.5) in the special form

$$\begin{aligned}
& d_N^{n-j}\mathbf{h}_{j,\,n-j} = \mathbf{h}_{n,\,0} + \sum_{k=j+1}^{n} d_N^{n-k}\boldsymbol{\varphi}_{k-1,\,n-k}, \\
& d_N^{-j}\mathbf{h}_{j,\,n-j} = \mathbf{h}_{0,\,n} - \sum_{k=1}^{j} d_N^{-k}\boldsymbol{\varphi}_{k-1,\,n-k} \\
& D_N^{n-j}\mathbf{H}_{j,\,n-j} = \mathbf{H}_{n,\,0} + \sum_{k=j+1}^{n} D_N^{n-k}\boldsymbol{\Phi}_{k-1,\,n-k}, \\
& D_n^{-j}\mathbf{H}_{j,\,n-j} = \mathbf{H}_{0,\,n} - \sum_{k=1}^{j} D_N^{-k}\boldsymbol{\Phi}_{k-1,\,n-k} \\
& (\Phi_{i,\,j} = \partial^{i+j}\Phi/\partial\alpha^i\,\partial\tau^j)
\end{aligned} \tag{3.6}$$

After multiplying the first and third equations by the vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\Lambda}$, respectively, and utilizing the equation $\mathbf{h}_{0,n} = \mathbf{H}_{0,n}$ which holds for $\alpha = 0$ by virtue of (2.6), we obtain a system of two equations to determine the derivatives $\mathbf{H}_{j,n-j}$ on γ_0 in terms of derivatives of lesser overall order in the variables α and τ

$$\begin{aligned}
& \boldsymbol{\lambda} d_N^n D_N^{-j}\mathbf{H}_{j,\,n-j} = \boldsymbol{\lambda}\mathbf{h}_{n,\,0} + \boldsymbol{\lambda}\sum_{k=1}^{n} d_N^{n-k}\boldsymbol{\varphi}_{k-1,\,n-k} - \boldsymbol{\lambda}\sum_{k=1}^{j} d_N^n D_N^{-k}\boldsymbol{\Phi}_{k-1,\,n-k} \\
& \boldsymbol{\Lambda} D_N^{n-j}\mathbf{H}_{j,\,n-j} = \boldsymbol{\Lambda}\mathbf{H}_{n,\,0} + \boldsymbol{\Lambda}\sum_{k=j+1}^{n} D_N^{n-k}\boldsymbol{\Phi}_{k-1,\,n-k}
\end{aligned} \tag{3.7}$$

Eqs.(3.7) are solvable in the case of linear independence of the vectors $\boldsymbol{\lambda} d_N^{\,n}$ and $\boldsymbol{\Lambda} D_N^{\,n}$ for each natural n. On satisfying the solvability conditions for (3.7), the derivatives of the remaining functions are determined fairly simply from the remaining equations of the system and the boundary conditions. It is here convenient to write the third boundary condition of (2.2) in the following form, solved for s, S:

$$s = f_2(p,\, p_1,\, v_1), \quad S = F_2(P,\, p_1,\, v_1)$$

Lemma. The solvability condition for (3.7) is satisfied at points of the set $\omega_e \subset \omega_1$ characterized by the following property: the normal velocity γ_{0t} relative to the gas behind reflected shocks is greater than the local velocity of sound ($|\mathbf{q}| > |\mathbf{k}|\, c$ for $\mathbf{x},\, t \in \omega_e$).

Proof. We reduce the matrices d_N and D_N to diagonal form

$$\begin{aligned}
& d_N = \varkappa^{-1} d_1 \varkappa, \quad D_N = \chi^{-1} D_1 \chi \\
& d_1 = \operatorname{diag}(\nu_1,\, \nu_2), \quad D_1 = \operatorname{diag}(N_1,\, N_2) \\
& \varkappa = \frac{1}{\sqrt{2}}\begin{Vmatrix} 1, & -z \\ 1, & z \end{Vmatrix}, \quad \chi = \frac{1}{\sqrt{2}}\begin{Vmatrix} 1, & Z \\ 1, & -Z \end{Vmatrix} \\
& \nu_{1,\,2} = J\left(1 \pm \left(\frac{|\mathbf{q}|^2 c^{-2} - |\mathbf{k}|^2}{|\mathbf{q}|^2 v_n^{-2} - |\mathbf{k}|^2}\right)^{1/2}\right)_{N,\,\Omega_1}, \\
& N_{1,\,2} = J\left(1 \pm \left(\frac{|\mathbf{q}|^2 c^{-2} - |\mathbf{k}|^2}{|\mathbf{q}|^2 v_n^{-2} - |\mathbf{k}|^2}\right)^{1/2}\right)_{N,\,\Omega_2} \\
& z = (|\mathbf{m}|\,\rho^{-1}|\mathbf{q}|^{-2}(|\mathbf{q}|^2 c^{-2} - |\mathbf{k}|^2)^{1/2})_{N,\,\Omega_1}, \\
& Z = (|\mathbf{m}|\,\rho^{-1}|\mathbf{q}|^{-2}(|\mathbf{q}|^2 c^{-2} - |\mathbf{k}|^2)^{1/2})_{N,\,\Omega_2}
\end{aligned} \tag{3.8}$$

Here f_{N,Ω_i} denotes limit values from the domains Ω_i for the quantity f at the point N. By using these formulas the solvability condition can be represented in the form

$$\begin{aligned}
& (N_1 N_2^{-1}\nu_1\nu_2^{-1})^n - A_1 H_1 (\nu_1\nu_2^{-1})^n - a_1 h_1 (N_1 N_2^{-1})^n - a_1 A_1 \neq 0, \quad n = 1,\, 2,\, \ldots \\
& A_1 = (\Lambda - Z)(\Lambda + Z)^{-1}, \quad a_1 = (\lambda + z)(z - \lambda)^{-1}, \quad H_1 = -h_1 = (z - Z)\times(z + Z)^{-1}
\end{aligned} \tag{3.9}$$

By virtue of the Zemplen theorem /6/ $|v_n| < c$ behind the shock, consequently, $\nu_1 > \nu_2 > 0$, $N_1 > N_2 > 0$ for $|\mathbf{q}| > |\mathbf{k}| c$. It follows from (2.9) that $\Lambda > 0, \lambda < 0$ (the monotonicity of the (p, σ)-pattern), and then $|a_1| < 1, |A_1| < 1, |H_1| < 1$. Therefore, the left-hand side of (3.9) grows as n increases, but is positive for $n = 0$: $1 - A_1H_1 - a_1h_1 - a_1A_1 > 0$. The lemma is proved.

We will show that condition (3.9) is not satisfied in the general case at those points $x, t \in \gamma_0$ where the normal velocity γ_{0t} with respect to the gas is less than the velocity of sound on at least one side of the contact discontinuity Γ_5. If this holds in Ω_1, then ν_1, ν_2 are complex conjugate quantities and z is purely imaginary. We set $\nu_1\nu_2^{-1} = e^{i\psi}$, $H_1 = e^{i\varphi}$, $a_1 = e^{i\delta}$ then (3.9) is converted to the form

$$e^{in\psi} \neq -e^{i(\varphi+\delta)}[(N_1N_2^{-1})^n - A_1e^{-i\varphi}][(N_1N_2^{-1})^n - A_1e^{i\varphi}]^{-1}$$

For fixed n equality is achieved here for $\psi_k, \varphi_k, \delta_k$ satisfying the relationships $\xi_{nk} = 0$ $(k = 0, 1, \ldots, n-1)$, where

$$\xi_{nk} = \psi - \left(\frac{2k\pi}{n} + \frac{\pi + \varphi + \delta}{n} + B_n\right); \quad B_n = \frac{1}{n}\arg\frac{(N_1N_2^{-1})^n - A_1e^{-i\varphi}}{(N_1N_2^{-1})^n - A_1e^{i\varphi}} \tag{3.10}$$

Let $\psi(\beta, \gamma)$ vary between $\psi_0 - \varepsilon_1$ and $\psi_0 + \varepsilon_1$ as the parameters $\beta, \gamma(\beta)$ vary along a certain path $(\beta \in [\beta_0 - \varepsilon_2, \beta_0 + \varepsilon_3], \varepsilon_i > 0, i = 1, 2, 3)$ and for integer $p, q, p < q, \psi_{10} = 2\pi p/q \in [\psi_0 - \varepsilon_1, \psi_0 + \varepsilon_1]$. Then for $k = ps, n = qs$ and sufficiently large s we have $\xi_{nk}(\beta_0 - \varepsilon_2) < 0$, $\xi_{nk}(\beta_0 + \varepsilon_3) > 0$. By continuity, the ξ_{nk} vanish at intermediate points of the path mentioned. Therefore, in the "subsonic" case the problem has no piecewise-analytic solution for general data.

In the "supersonic" case, all the derivatives of the solution can be found uniquely on γ_0 at an arbitrary point $N \in \omega_c$ according to the lemma, and a formal representation of the solution can be written down in the form of Taylor series. Convergence of the series is proved by constructing a majorant in the neighbourhood of the point N. The linear changes of variables

$$\mathbf{r} = \binom{l}{r} = \varkappa\mathbf{h}, \quad \mathbf{R} = \binom{L}{R} = \chi\mathbf{H}$$

reduce the matrices d_N and D_N in (3.5) to diagonal form

$$d_1\mathbf{r}_\tau = \mathbf{r}_\alpha + \boldsymbol{\psi}, \quad D_1\mathbf{R}_\tau = \mathbf{R}_\alpha + \boldsymbol{\Psi} \tag{3.11}$$
$$\boldsymbol{\psi} = \binom{\psi_1}{-\psi_2} = \varkappa\boldsymbol{\varphi}, \quad \boldsymbol{\Psi} = \binom{\Psi_1}{-\Psi_2} = \chi\boldsymbol{\Phi}$$

The boundary conditions are reduced to homogeneous conditions by standard substitutions (we retain the notation of the quantities after the substitution). Then for $\tau = 0$ and $\alpha = 0$

$$(L - A_1R)|_{\tau=0} = (l - a_1r)|_{\tau=0} = (R - H_1L - k_1l)|_{\alpha=0} = 0 \tag{3.12}$$
$$(r - h_1l - K_1L)|_{\alpha=0} = 0 \quad (k_1 = 2Z(z+Z)^{-1}, \; K_1 = 2z(z+Z)^{-1})$$

The solution of the transformed Eqs (3.7) is written down explicitly

$$R_{j,n-j} = N_2^{j-n}\nu_2^{-n}\Delta_n^{-1}\Big\{b_n\sum_{k=j+1}^{n} L_k(\Psi) + k_1N_1^n\sum_{k=1}^{n} l_k(\psi) + N_1^n\sum_{k=1}^{j}(b_nN_1^{-k}(\Psi_1)_{k-1,n-k} + c_nN_2^{-k}(\Psi_2)_{k-1,n-k})\Big\} \tag{3.13}$$
$$L_{j,n-j} = N_1^{j}\nu_2^{-n}\Delta_n^{-1}\Big\{c_nN_2^{-n}\sum_{k=j+1}^{n} L_k(\psi) + A_1k_1\sum_{k=1}^{n} l_k(\Psi) + A_1\sum_{k=1}^{j}(b_nN_1^{-k}(\Psi_1)_{k-1,n-k} + c_nN_2^{-k}(\Psi_2)_{k-1,n-k})\Big\}$$
$$\Delta_n = (N_1N_2^{-1}\nu_1\nu_2^{-1})^n - a_1h_1(N_1N_2^{-1})^n - A_1H_1(\nu_1\nu_2^{-1})^n - a_1A_1(k_1K_1 - h_1H_1)$$
$$b_n = \nu_1^nH_1 + a_1(k_1K_1 - h_1H_1)\nu_2^n, \quad c_n = \nu_1^n - a_1h_1\nu_2^n$$
$$L_k(\Psi) = N_1^{n-k}(\Psi_1)_{k-1,n-k} + A_1N_2^{n-k}(\Psi_2)_{k-1,n-k}$$
$$l_k(\Psi) = \nu_1^{n-k}(\psi_1)_{k-1,n-k} + a_1\nu_2^{n-k}(\psi_2)_{k-1,n-k}$$

The formulas for the derivatives $r_{j,n-j}, l_{j,n-j}$ are analogous in form and can be obtained from (3.13) by a formal replacement of the lower-case letters by upper case and conversely.

The formulas presented show that the derivatives being determined will increase as the coefficients of the derivatives ψ_i, Ψ_i increase, and by the replacement of ψ_i, Ψ_i by their majorizing functions. Taking this fact into account, a problem is constructed to determine the majorant: the A_1, a_1, H_1, h_1, K_1 in the boundary conditions (3.12) are replaced by their absolute values, while k_1 is replaced by $k_2 = |a_1A_1K_1|^{-1}(1 - |a_1h_1|)(1 - |A_1H_1|)$. If such substitutions are made in the expression for Δ_n, then the inequality $\Delta_n \geqslant \Delta_{n2} > 0$ is satisfied for the quantity Δ_{n2} obtained in the result.

The inequalities

$$|b_n| < M_1(\nu_1^n|H_1| + \nu_2^n|a_1|(k_2|K_1| - |h_1H_1|); \quad c_n < M_1(\nu_1^n - |a_1h_1|\nu_2^n)$$
$$M_1 = \max\,(1,\; (|H_1| + |a_2|)|A_1|(1 - |a_1h_1|)^{-1},\; (1 + |a_2h_2|) \times (1 - |a_2h_2|)^{-1})$$

hold for the coefficients b_n and c_n in (3.13)

Consequently, if the equations to determine the majorants of r, l, R, L are taken in the form of (3.11), by replacing ψ_i and Ψ_i by $M_2\psi_{im}, M_2\Psi_{im}$ (where ψ_{im}, Ψ_{im} are the majorants of $\psi_i, \Psi_i, M_2 \geqslant \max(|k_1| \times k_2^1, M_1)$), then the derivatives of the majorants that are being determined successively will not be less than the absolute values of the derivatives of the solution. The systems of coefficients for the derivatives in the remaining equations are replaced by their majorants (Eqs.(3.1) reduce to quasilinear by using differentiation).

The majorants of ψ_{im}, Ψ_{im} can be selected so that the following relationships are satisfied

$$\Psi_{1m} = (1-\mu N_1^{-1})(\mu N_2^{-1}-1)^{-1}|A_i|\Psi_{2m} = k_2|a_1A_1|(1-\mu N_1^{-1})(\mu N_2^{-1}- 1)^{-1}(1-|A_1H_1|)^{-1}\psi_{2m}, \quad \psi_{1m} = |a_1|(1-\mu\nu_1^{-1})(\mu\nu_2^{-1}-1)^{-1}\psi_{2m}; \ \mu = (J)_N \tag{3.14}$$

This is achieved by selecting the common majorants of all ψ_i, Ψ_i multiplied by a sufficiently large numerical coefficient as ψ_{2m}.

Relations (3.14) enable us to seek a particular solution of the majorant problem that satisfies the relationships

$$L = |A_1|R, \ l = |a_1|r, \ R = |H_1|L + k_2 l, \ r = |h_1|l + |K_1|L$$

According to the well-known properties of analytic functions, majorants of the coefficients for the derivatives in the equations can be selected so that the independent variables will enter in the form of a linear combination $\xi(\tau+\mu\alpha)+\beta-\beta_N+\gamma-\gamma_N=\eta, \ \xi \geqslant 1$. This enables us to seek the particular solution of the majorant problem in the class of functions dependent only on η. The system of ordinary differential equations obtained is reduced to normal form by an appropriate selection of the parameter ξ. The existence of an analytic solution of the Cauchy problem with homogeneous data for $\eta = 0$ for a system of ordinary differential equations follows from the Cauchy-Kovalevskaya theorem. Convergence of the series governing the solution as a function of the auxiliary variables $\tau, \alpha, \beta, \gamma$ is thereby proved. If the coefficients A_1 or a_1 vanish, small changes occur in the proof (see Sect.4).

4. Configurations of a shock and centred wave. Let the intersection of the (p, σ)- pattern determine the configuration of a shock and centred wave on γ_0. In conformity with Sect.2, the amplitude of the centred wave (CW) on γ_0 is known, hence, the CW adjoining the given solution is found independently /3/. The problem of constructing the solution in a domain bounded by a closing characteristic of the CW and the contact discontinuity as well as in a domain bounded by the shock surface and the contact discontinuity differs from that considered above by the fact that conditions of continuous abutment to the CW should be satisfied on Γ_4 which is a sonic characteristic. The remaining boundary conditions do not change. New variables can be introduced in the domain Ω_2, bounded by Γ_4 and Γ_5 by using (3.1) in which we put $v_n = c$.

After the change of variables, (1.1) take a form analogous to (3.3), here

$$D = \frac{(\mathbf{x}_\tau - \mathbf{x}_\alpha)\mathbf{q}}{|\mathbf{q}|^2}\begin{Vmatrix} (\zeta\mathbf{q}) & (1-c^2|\mathbf{k}|^2|\mathbf{q}|^{-2})\rho^{-1}c^{-2}J|\mathbf{m}| \\ \rho|\mathbf{q}|^2 J|\mathbf{m}|^{-1} & (\zeta\dot{\mathbf{q}}) \end{Vmatrix}$$

(in the notation of the fundamental quantities in the domain Ω_2 the lower-case letters have been replaced by upper case). On the boundary $\tau = 0$, the $A, B, S, \Sigma + H$ are given as functions of the variables α, β, γ. By analogy with (3.4) it is convenient to write this last condition in the form

$$\mathbf{\Lambda}\begin{pmatrix}\Sigma \\ P\end{pmatrix}\bigg|_{\tau=0} = F_1(P, \alpha, \beta, \gamma)|_{\tau=0} \tag{4.1}$$
$$\mathbf{\Lambda} = (1, \Lambda) = (1, (\partial H/\partial P)_N), \quad (\partial F_1/\partial P)_N = 0$$

Taking account of the formal similarity between the problem obtained and the problem examined in Sect.3, further construction of the solution is carried out by the same scheme. The specific features of the problem are associated with the fact that the surface Γ_4 is a characteristic, consequently, the matrix D is degenerate, its eigenvalue is $N_2 = 0$. Moreover, $A_1 = 0$. Consequently, the condition for the continued equations in the derivatives to be solvable has the following form here:

$$(\nu_2\nu_1^{-1})^n - a_n h_1 \neq 0, \quad n = 1, 2, \ldots$$

This condition is satisfied in the supersonic case and the general case and is not satisfied in the subsonic case. A representation of the solution in the form of Taylor series can be written down when the solvability condition is satisfied.

A certain distinction from Sect.3 occurs in the proof of the series convergence in that $A_1 = 0$. The vector $\mathbf{R}$ is introduced by the same formulas as in Sect.3. After conversion of the boundary conditions to homogeneous conditions, Eq.(4.1) goes over into $L=0$ for $\tau=0$. In the majorant problem the coefficients $a_1, A_1, h_1, H_1, k_1, K_1$ of the boundary conditions (3.12) are replaced by their absolute values. For an appropriate selection of the majorants ψ_{im} Ψ_{im} for the analogues of (3.11), the particular solution of the majorant problem satisfies the relationships

$$r=\frac{|K_1|}{1-|a_1h_1|}L,\quad l=\frac{|a_1K_1|}{1-|a_1h_1|}L,\quad R=\left(|H_1|+\frac{|a_1k_1K_1|}{1-|a_1h_1|}\right)L$$

everywhere in the domain of definition. The existence of a piecewise-analytic solution is thus proved for the shock and centred wave configurations in the case when the normal velocity γ_{0t} relative to the gas behind the shock is greater than the velocity of sound (the mentioned velocity is always subsonic in the domain behind the CW).

It is necessary to reverse the replacement of the dependent and independent variables to prove the existence of analytic solutions of system (1.1) describing the interaction of strong discontinuities. The vector $\mathbf{u}$ is restored by using relations of the type (2.10). The local reversal of the change of variables $t=\tau+\alpha+t_0(\beta, \gamma)$, $\mathbf{x}=\mathbf{x}(\tau, \alpha, \beta, \gamma)$ is possible because of the non-degeneracy of the appropriate Jacobians. Proof of the existence in the space $R^4(\mathbf{x}, t)$ of a neighbourhood of the set ω_c possessing the same property, as the mapping $\mathbf{x}, t\to(\tau, \alpha, \beta, \gamma)$ univalent by in the mentioned neighbourhood is analogous tc the proof of the corresponding facts in /3, 4/. Hence, we have proved the following theorem.

Theorem. A piecewise-analytic solution of (1.1) exists describing the interaction of strong discontinuities defined in a certain neighbourhood of the set ω_c in $R^4(\mathbf{x}, t)$.

The solution constructed describes the gas flow, the shock fronts, the contact discontinuities, and the CW in the neighbourhood of the lines of intersection of interacting fronts γ_{0t} moving in $R^3(\mathbf{x})$. In the initial period of the interaction the normal velocity γ_{0t} relative to the gas behind the reflected waves is greater than the velocity of sound ($|\mathbf{w}|\to\infty$ in the neighbourhood of the point Q as $t_\beta^2+t_\gamma^2\to 0$). As has been proved, the analyticity of the solution is conserved here for analytic data. The solvability conditions of the problem are not satisfied in the class of analytic functions for a subsonic normal velocity γ_{0t}. This means the appearance of singularities in the solution. Transfer to a non-regular interaction of discontinuities is possible at a later stage. Study of the transition process requires additional examination.

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